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ABSTRACT

The identification of teaching resources for low achievers depends upon analysis of student characteristics, teaching strategies, and mathematics curriculum content. Social and emotional problems of low achievers are often reflected in low motivation, short interest spans, antisocial behavior, high absence rates, and goals based only on the immediate future. These characteristics suggest that appropriate resource materials should be constructed in small units with provisions for individualized work, and should emphasize success and self-worth--possibly by including an incentive or reward system. An activity learning approach provides an excellent vehicle for promoting the following teaching procedures that are particularly effective with low achievers: (1) Employ repetition through spiraling; (2) Introduce small segments of material at a time; and (3) Use language which the class is likely to understand. Resource materials should have clear objectives which have some relationship to the immediate needs of the learner. Content material should develop desirable habits, attitudes, and appreciations as well. To do this, materials must combine low difficulty levels with high mathematical standards. (Author/CK)

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Putting Research into Educational Practice

PREP Report No. 30

Teaching Resources for Low-Achieving Mathematics Classes

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U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE/Office of Education
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Teaching Resources for Low-Achieving Mathematics Classes

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Columbus, Ohio

1972

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TEACHING RESOURCES FOR LOW-ACHIEVING MATHEMATICS CLASSES

The dust generated by the mathematics curriculum reform movement of the 1960's has settled, and some definite statements concerning the current scene can now be offered. First, it is clear that the curriculum has been submitted to a thorough-going revision. Mathematical content has been radically upgraded, and grade levels at which it is presented have been lowered. More and better mathematics has been made available to students at an earlier age.

By and large, however, the majority of students in our schools have not been a party to the proclaimed benefits attending the mathematics reform movement. While the more mathematically able students have gleaned much from newer curriculums, the less able students have found little which meets their needs, appeals to their interests, or is accessible within their conceptual framework. Carl B. Allendoerfer has remarked that the first revolution, the new mathematics movement of the last decade, was "guided by college people and was largely aimed at the college-capable student." The emerging need now is for the new types of structures which are more properly suited to the remaining seven-eighths of the school population, and particularly to the lower one-third. This second revolution will be initiated when serious attention is paid to the growing body of knowledge on how young children learn mathematics. (91)

Few would contend that a "second revolution" in school mathematics has now been set in motion. But there are indications, at least, that ground work for a genuine reformation is being laid. For example, there appears to be a growing point of view that, while the first reform movement started with a body of knowledge to be taught and attempted to fit the students into it, a more successful and pedagogically sound approach is to begin with the student; assess his learning styles, interests, and talents; and then search for subject matter and instructional strategies which appear to be appropriate to him at his particular stage of development. As Jean Piaget (30), who is having great impact on current thinking in curricular matters, has remarked:

> It is a great mistake to suppose that a child acquires the notion of number and other mathematical concepts just from teaching. On the contrary, to a remarkable degree he develops them himself, independently and spontaneously. When adults try to impose mathematical concepts on a child prematurely, his learning is merely verbal; true understanding of them comes only with his mental growth.

In a prophetic article some 5 years ago which has proved to be remarkably accurate, Davis (100) outlined four changes in elementary school mathematics which he saw as "most urgent":

1. Greater use of physical materials in mathematics classes.
2. Greater diversity of types of experiences in the child's day.
3. Identification and early introduction of basic mathematical ideas.
4. More emphasis on student originality and creativity within the school mathematics program.

These four needs have received increased attention and they are now considered particularly important to the low-achieving child not only in the elementary school, but also throughout his formal educative experience.

This report puts together a collection of ideas, practices, and techniques which have been used with low achievers in recent years. Although there has, of course, been an element of judgment used in selection, as far as possible decisions as to the effectiveness of a particular teaching resource are left to the individual. Such a policy appears to be consonant with the complexity of the teaching task presented by the low achievers. So many factors come into play in a given situation, and so many decisions have to be made by the teacher during any one lesson, that little predictive power exists at present concerning the effectiveness of particular strategies and materials.

The challenge of teaching children having learning difficulties in mathematics is great. But so are the rewards. Another curricular "happening" during the 1960's, made possible by extensive funding by the Federal Government and other agencies, was that of special projects devoted to low achievers and their problems. Over 40 such projects were identified by the Committee on Mathematics for the Noncollege Bound, appointed by the National Council of Teachers of Mathematics. And one message from these groups is loud and clear: Once a teacher has managed to generate a classroom situation highly conducive to learning mathematics and has witnessed the responses of "slower" students to these situations, remarkable things happen. Enthusiasm grows, spirits lift, and low-achieving classes become an opportunity and challenge for both students and teachers. Many of the innovations and points of view of the projects for low achievers are reflected in the review of teaching resources which follows.

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CHARACTERISTICS OF LOW ACHIEVERS

The literature abounds with lists of characteristics of students who have difficulties in learning mathematics. (See, for example, Hoffman (19)). Most such lists can be divided into two sections: characteristics which arise from social and emotional problems, and characterlistics arising from learning difficulties or achievement failures in mathematics.

Social and emotional problems are often reflected in the following characteristics of low-achieving students:

- High rate of absence
- Goals based on a view of the immediate future
- Low motivation
- Antisocial behavior
- Short interest span
- Inability to see the practical uses of mathematics

Sobel (41) focuses particularly upon the slow learner in junior high school, observing that:

- These are the years of rapid, uneven growth and there is need for both teachers and students alike to understand this growth and to realize that it is both variable and individual in nature.
- Youngsters of this age, continuing a drive which begins at birth, seek personal independence from both parents and teachers. They seek peer acceptance and have a need to belong to some social group.
- They are insecure, primarily due to the tremendous physical changes taking place at this age, and they crave security and success.
- They want recognition, approval, and status.
- Their interests change rapidly; they crave new experiences while at the same time longing for the security of the old.

He then summarizes:

> . . . the slow learner in the junior high school has the same characteristics as other pupils of the same age; the same basic needs and interests. However, more than the average child, he needs to be given the chance to experience success and approval; more than the others he needs to feel that he is a member of a group with a contribution to make; he needs status; his confidence must be re-established; his interest stimulated; his attitude toward mathematics made favorable; his ego flattered.

In an address to a 1963 conference on the low achiever in mathematics, Henry (18) commented that homes of low achievers are physically and personally disorganized, life does not run on a time schedule, and so on. Thus, emotionally and cognitively, they lack the structure on which a conventional educational system can build. He suggests that these students need emotional calming down before any attempt at work, and recommends that the student to teacher ratio should be between 10 and 15 to 1, and that additional supernumerary personnel should be available.

Low achievers are often equated with disadvantaged students, but Sasse (123) points to distinctions which should be made between various categories of students who are behind grade level in mathematics. She notes, for example, that the "unwilling, psychological dropout has not lagged behind for the same reasons as has the slow learner," and further comments that "the culturally deprived responds to different experiences than does the emotionally starved."

Regardless of the classification or label assigned to a student or group of students, the critical factor lies in the point of view assumed by the teacher toward these students. Robert Davis (5), writing some 15 years ago on the role of emotions and attitudes in teaching, commented that:

Good teaching cannot be built on the attitude that students are stupid. . . . The good teacher builds on the correct part, whereas the poor teacher attacks the weaker part. Hence, the good teacher leads the student along the path to self-reliance and self-confidence, while the poor teacher induces feelings of inadequacy and distaste for the subject. As a sheer guess, we might estimate that the teacher is well advised to devote one-tenth of his effort to the objective presentation of material (which should be an easy task for him, anyway, if he is well qualified) and to spend the remaining nine-tenths in establishing a suitable emotional background for the students' efforts.

Many "disadvantaged" children have built up a wall of indifference concerning the value of school to them. Suggestions for breaking down this wall include each teacher developing the feeling that each student is important and is capable of learning; and working with small groups of children, rather than with the class as a whole, using games and puzzles, manipulative devices, and other interesting activities. For each child, a personal folder containing all written work, test papers, ditto work and any other items of interest may be prepared and kept up to date (11).

A concerted effort to provide materials which take account of the social and emotional characteristics of low achievers in upper elementary and junior high school is the writing project of The National Council of Teachers of Mathematics, "Experiences in Mathematical Ideas" (EMI) (159). The introduction to these materials states:

> The EMI project materials are designed so that teachers may implement them in conventional self-contained classrooms as well as in other organizational patterns—for example, in team teaching, multiunit programs, nongraded structures. Hence EMI materials are dependent only on the willingness of teachers to offer pupils opportunities and freedom to learn those basic mathematics concepts that have been provided. Teachers are a crucial and central element in implementing student-centered and activity oriented materials. They must be exceptionally kind, warm, and fair, displaying a sincere empathy for pupils with learning problems.

We must help the slow learner develop faith in himself. He must hope for the future and confidence in his ability if he is to experience any degree of success (120).

The first factor in a security-giving relationship is a sincere valuing of the child by the teacher. This means having an inner conviction that the child has good in him, has potentialities that can be realized, and is worth the thought and effort involved in helping him (32).

Incentives and rewards are being tried in more and more low achiever programs as a basis for increasing the student's confidence in his abilities. Preliminary feedback from performance contracting projects in mathematics funded by the Office of Economic Opportunity indicate that most of these programs use incentive rewards to obtain improved academic performance by low achievers. The initial contractor in this area, Dorsett Educational System used a reward under which students received green stamps, transistor radios, and chances on a color TV for academic improvement (33).

A program operated by the Teacher Corps to train interns from St. Cloud College also uses incentive methods with nonachievers in Minneapolis junior high schools (33). The report states:

> The program is designed to improve educational opportunities for poor children and to attract good teachers to schools in poor neighborhoods. Techniques used by the interns to help motivate their pupils include: Paying them for school attendance by giving them credit to purchase items at a local store; deducting for tardiness or rowdiness from these earnings; encouraging students to participate in planning and conducting classes, allowing each student to select 10

books for his school library; providing personalized letterheads to students to encourage letter writing. Twenty-four college-graduate interns with noneducation degrees work in teams of four to six under an experienced teacher.

In contrast to external motivation of low achievers by rewards and incentive methods, Allen (46) makes a case for self-motivation through the curriculum itself. He argues that the important factors are the intricacies and the effects upon motivation that can be achieved by appropriately designed materials for arranging a learning situation in which participants are actively responding to challenging problems constructed by themselves in a way that is both enjoyable and self-paced. Self-motivated learning is the lofty goal of the efforts to construct such autotelic learning situations. If such techniques ultimately turn out to be applicable in some manner to the learning of other skills, can anyone reasonably doubt the usefulness of such efforts to the educational profession?

Fremont and Ehrenberg (12) have had success in motivating low achievers by focusing upon "the use of the language of algebra to describe relationships in the world in which we live. As a result of their work with ninth graders, they suggest that "skill in carrying out fundamental operations and ability in mathematics may not be one and the same." Attempts to change the students' attitudes of incompetency and inadequacy in mathematics were made by showing them that they could do the same mathematics as the other students.

In a pilot study on 38 low achievers in the Parish Schools in Lafayette, Louisiana, partially programed mathematics material and mathematical games were used in an attempt to improve mathematical achievement and attitudes toward mathematics of the students involved. Results of the pilot study indicated that the modified programed lecture and mathematical games instructional approach was effective in teaching ninth grade general mathematics. The effectiveness was reflected both in the achievement scores and in the evaluation of the attitude data (63).

Because much of school mathematics is a hierarchy of prerequisite skills, concepts, and principals, many lists of low-achiever characteristics typically include such items as:

- A record of failure in mathematics and a fear of the subject
- Achievement scores at least 2 years below grade level
- Reading and comprehension difficulties in many cases
- Inability to follow directions well
- Tendency to leap to conclusions
- Inability to generalize

Following are the results in a study (34) conducted on sixth- and seventh-grade students who were 2 years below their grade level in arithmetic achievement:

- Mean percents of accuracy in the basic computational processes were as follows: addition, 88 percent; subtraction, 79 percent; multiplication, 60 percent; division, 33 percent; addition of fractions, 18 percent; subtraction of fractions, 14 percent; multiplication of fractions, 8 percent; division of fractions 2 percent.
- Students evidenced satisfactory reasoning in word problems involving addition and subtraction of whole numbers, but made frequent reasoning errors in problems involving multiplication and division of whole numbers as well as in all the processes involving common fractions.
- Sixteen (out of 20) of the students were one or more years below their mental grade level in functional reading ability as measured by the **Durrell Analysis.**
- Teacher assessment and cumulative records indicated that students were underachieving generally in school subjects other than arithmetic.
- Teacher assessment and personality tests indicated that students characteristically were withdrawn and defeated in their attitudes toward school and society.

- Sixty-three percent of the causes of underachievement identified by classroom teachers were of an emotional nature, involving lack of interest, home or school maladjustment, short attention span, or limited initiative.
- Fifteen of the students had shown immaturity or slowness of general development, while 13 manifested abnormal physical conditions, ranging from low vitality to rheumatic fever.
- Parents tended to be from lower socioeconomic classes. Three parents owned small businesses, two were salesmen, 12 were skilled or unskilled laborers, and three were unemployed.
- Parents of 12 of the students tended to hold one or more teachers responsible for their child's inadequacies.

Homan (21) has discussed five steps for a diagnostic remedial process, as outlined by Bateman (1), and then suggests methods and aids which may help the child with a learning disability in arithmetic. The five steps dealt with are:

1. Determine whether a learning problem exists.
2. Obtain a behavioral analysis and description of the disability.
3. Study the relevant correlates of the disability.
4. Formulate (done by diagnostician) a clear, concise, and accurate diagnostic hypothesis.
5. Make educational recommendations.

Homan deals with the following difficulties associated with learning disabilities in arithmetic; lack of perceptual skills, motor disinhibition, perseverance, language disabilities, low level of abstract thinking. She then outlines specific remedial techniques for dealing with the various difficulties. For example, a child with motor disinhibition (inability to coordinate and integrate his visual, auditory, and motor activity) may be helped by counting on a magnetic board or felt board, or by use of class activities involving identification and counting.

An article by Sarah Greenholz (16) is directed to the teaching of pupils who rank below the 30th percentile of the student population in mathematical achievement. Some interesting and helpful suggestions are presented in the areas of problems with underachievers (e.g., "How do you grade them?"), general techniques which have been used successfully in the classroom (e.g., "Give these pupils immediate satisfaction by checking their work as they do it."), techniques in mathematics which also have been used successfully in the classroom (e.g., "fold a strip of newspaper to represent 50, 25, 12½, 33⅓ and 16⅔ percent of the whole"), and finally to increase the teacher's competence at this level (e.g., "Have some exchange sessions with other teachers on what has worked in your classroom").

The following teaching suggestions summarize the philosophy of many of the more promising directions in working with low achievers:

- Accept children as they are. Like them as they are so they can like themselves.
- Show a faith in the child that enables the child to have faith in himself.
- Make him feel that it is all right to try. If he fails, failure is not a crime.
- Be pleased with a reasonably good attempt.
- Show confidence in his ability to become competent (7).

ACTIVITY LEARNING FOR LOW ACHIEVERS—INCLUDING USE OF CALCULATORS AND COMPUTERS

The difficulties encountered by low achievers in mathematics classrooms have been attacked on a massive scale by the use of computational aids and by involving the student in learning activities where he is an active participant. The Office of Education report on the **Low Achiever in Mathematics** found that common elements in programs for the low achiever throughout the country are:

- A mathematics laboratory—whether it be a center for the school, a formal laboratory for the use of a few selected classes, or a classroom laboratory.
- The use of calculators to help the student find his pattern or error in computation, and to enable him to get past simple computational blocks to basic mathematical understandings.
- A regulated program with a pattern of activities for security, but with a change of activities to accommodate the short attention-span of the slow learners and the unit-a-day pattern for the satisfaction of a task completed and evaluated on the spot.
- Provision for reinforcement of early basic concepts, which may be weak, utilizing the method and techniques of the more modern programs in mathematics—the exploration of the structure of the number system, the experimentation and discovery of patterns, and their utilization.
- The use of many manipulative devices, such as the abacus, cuisenaire rods, geoboards, etc.
- The proper and controlled use of games, puzzles, and other motivational techniques.
- The use, where possible, of remote terminals tied into computers for computer-aided instruction units (45).

The report also made specific recommendations for beginning teachers of low achievers:

- Learn as much mathematics as you can. You must know mathematics unusually well to be able to split up the usual processes into simple elements.
- Study the recent developments in the curriculum and acquire experience in teaching the new courses. You should know the best mathematics in the best expositions available before you can adapt these ideas to the special needs of low achievers.
- Request from your administration an adequate supply of manipulative materials, and learn how to use them in teaching low achievers.
- Become acquainted with what these students are doing in their industrial arts, business education, and home economics classes, and use these courses as sources of applications of mathematics.

Computational Aids

The School Mathematics Study Group explored the use of charts and tables in assisting junior high school students to overcome the "barrier of computation" as they approached the subject matter of the regular SMSG curriculum (103, 104).

Four kinds of problems lend themselves quite well to calculator- (or computer-) of exponents (i.e., having students prepare tables of power of some number, say 5, and then answer a question such as "$5^3 \times 5^4 = ?$"); (2) an intuitive approach to Pythagoras' theorem; (3) work involving the evaluation of roots of numbers; and (4) work related to logarithms (88).

Stenzel (42) found that motivation of low achievers was greatly improved by the introduction of desk calculators and grocery store type computing scales into her classroom.

A report (126) in **Today's Education** gives an account of how in the Los Angeles and Kansas

City School systems computers are being used in mathematics instruction.

> . . . Eighth graders in Bingham Junior High School spend from 30 minutes to 2½ hours a week participating in an instructional dialogue on supplementary and enrichment topics in grade 8 science and mathematics. They use the IBM 1500 Instructional System.
>
> Computer and calculator firms in the Greater Los Angeles area have provided nearly $200,000 worth of equipment for experimental purposes at no cost to the schools. Without exception, the desk-top calculators (which can be programed) have been highly successful and are enthusiastically received by both the gifted students and the slow learners, many of whom were potential drop-outs. "Nothing turns them on like these calculators," claims the spokesman.
>
> Shoemaker (38) has developed materials employing the desk calculator in the high school classroom. He outlines three states of machine usage in the classroom: At the outset, the students are overwhelmed with the complexities of the machine, followed in a few weeks by enthusiasm and desire to master the "mechanical genius." With proper guidance, learning soon moves into a problem-solving and discovery-learning atmosphere, creating the desired result—using the calculator as a tool of mathematics.

Materials developed by teachers in Arkansas, called "Drop-In Mathematics" (106) involve exercises in which students use flow charting as well as printing calculators as aids in handling troublesome arithmetic in the solution of problems. The materials are written so that a unit per day may be taught. The teacher may cover sequential units on consecutive days or nonrelated units. Each unit has suggested procedures for presentation. The intended level for usage is ninth and tenth grade general mathematics.

Hoffman (19) has identified directions for special projects for the slow learner which involve use of a mathematics laboratory, with all of its ramifications, including calculators, remote terminals for computer, and flow-charting for problem analysis.

At the elementary school level, less work with computers and calculators appears to have been done to date. However, some have urged (see, for example, Travers and Knaupp (87) the early introduction of computational aids in the curriculum.

Teachers and administrators will find themselves faced with many decisions (budget, in-service training, sequencing of topics, for example) as they consider using calculators and computers in their schools. One technological development which is assuming growing importance is the emergence of programmable desk calculators. Such machines are able to execute rather elaborate series of instructions (programs) and in many cases may adequately serve the instructional needs of the students.

Laboratory Technique

Perhaps the most ardent proponent of the laboratory movement has been the English educator, Edith Biggs (53, 54). The philosophy of the laboratory approach to mathematics, together with an abundance of illustrations as to its use at various grade levels, elementary through junior high school, can be found in Biggs and Maclean's **Freedom To Learn** (54). The book advocates learning mathematics in an "active, creative way," with the following aims:

- To free students, however young or old, to think for themselves
- To provide opportunities for them to discover the order, pattern, and relations which are the very essence of mathematics, not only in the man-made world, but in the natural world as well
- To train students in the necessary skills.

Sections of the book are devoted to such topics as: location and use of equipment and materials, identification of reference books, use of audiovisual aids, role of the teacher in the classroom, and typical programs for teacher-training workshops.

May (71) describes the junior high learning laboratory in Winnetka, Illinois, as a place where students can explore, question and investigate, and where there is no failure. The slow learner finds that he can come to the laboratory to get help and gain strength in some area so that he can continue his work in the regular classroom.

Sweet (83) says that there are a number of reasons for operating a mathematics laboratory. One is that the laboratory method of teaching is a pedagogically sound technique. It utilizes an experimental approach that requires each student's participation, but allows him to work at his own rate. This leaves the instructor free to administer more individual help. Also, because the laboratory is conducted on a less formal level than the regular classroom, it provides an opportunity for more discussion among the students, and they learn from one another. Most of all, it adds variety as a change from the usual classroom routine. . . . The mathematics laboratory can be the focal point of interest–equally appealing to the slow learner and the capable student. It can be a time for field trips or group projects. It can provide the time for several students to investigate elementary group theory, or to make a filmstrip of optical illusions. One student might wish to use some of the laboratory periods to study computer programing. Another could calculate a value for pi using the Monte Carlo method. The class might study a unit on surveying, or produce a mathematical movie. Any of these activities will help students to develop mathematical concepts. However, their conclusions will be based on their own observations, rather than upon facts which were spoon-fed to them in a classroom lecture.

Two interesting features of the Oak Hill Elementary School Mathematics Laboratory program in Newton, Massachusetts, are the buddy system and the mathematics lab fair. Sixth-grade students volunteer and are then trained to help first-grade children who are either low achievers or are in need of some enrichment. The "buddies" work together for an entire school year, about 90 minutes per week. Both groups of children seem to profit from this arrangement.

The lab fair involves a large number of students working on a wide variety of mathematics projects, and then inviting their parents in for an evening of observation and participation (56).

Phillips (73) has reported teachers' reactions to working in a mathematics laboratory:

> In the mathematics laboratory, the teacher can individualize instruction, and get children to work voluntarily on a higher level of accomplishment than they otherwise would. Children can prove to themselves that what they are learning is correct and consequently rely less on authoritarian proof from teachers and textbooks.

Some of the larger goal-seeking projects which could be adapted for use with low achievers have to do with statistical studies in the school. Meyers (72) suggests, for example, devising tables and graphs of attendance, breaking the data down by classes, by weeks or months, correlating absence with weather, with boys and girls, with distance from school; displaying enrollment figures by showing distribution of the student body geographically; producing scale drawings of floor plans and rooms, maps of the school grounds; graphing budget figures of school overhead, data on library usage, fees and expenses of students, and average growth curves of students by grades, involving weights and heights, showing adolescent acceleration. Such a study could be summarized by committees of students, duplicated, bound in a report and distributed to the faculty, administration, and parents.

Hamilton (61), in attempting to help mathematics teachers develop discrimination in choosing and using manipulative devices, suggests the following characteristics of good devices:

- Outcome and organization must not be obscure.
- Variety must be provided.

- Devices must be simple to operate. (Remember that children often open a toy and then play with the box.)
- They should be simple to assemble and easy to store.
- Parts must not be easily lost.
- The device should not be an end in itself.
- The device should encourage communication of some sort.

In an article by Haggerty (60), a number of suggestions are given for mathematics projects at the junior high level. Typical are a marble device used to illustrate the normal distribution and a chart to show the relation of the laws of probability to genetics. While the article is not directed specifically to the low achiever, some ideas suggested might be appropriate for such students.

The Eighteenth Yearbook of the National Council of Teachers of Mathematics, **Multi-Sensory Aids in the Teaching of Mathematics,** includes many suggestions for activities which may prove to be effective for low achievers. Typical articles are: "Curve-Stitching in Geometry," by C.V. McCamman (69); "Window Transparencies," by J.A. Tennyson (85); and "The Mathematics Collection," by George Wolff (89).

Smart and Marks (78) give a partial listing of various inexpensive (under $10) weighing and measuring devices appropriate for elementary school pupils. They also give some sample exercises which would make use of these devices (e.g., "Place square inch tiles on the top of a shoe box until it is covered. How many inches does it take to cover the box?").

Fox (58) has suggested many opportunities for activities which may "make arithmetic more meaningful in the intermediate grades." Equipment inviting experimentation in computing and measuring includes: scales (both types), liquid measures, thermometers, erector sets, balloons, varied lengths of pipes, rulers, tuning fork, levers, pulleys, perceptual calendars, measuring cups and spoons, directional compass, compass for drawing circles, pedometer, map meter, stop watch, weights, clocks, flannel board & illustrations (fractional parts, magic square numbers, etc.), barometer, egg cartons, hour glass, sun dials, tape measures, metronome, toy piano, pitch pipe, catalog, toy money, chess, dominoes, monopoly (game), globe, and maps.

Activities which involve number, counting, computing, or measuring in various subjects include:

Physical Education

Marching by 2's, 3's, 4's
Rhythmic counts
Square dancing
Keeping score
Setting dimensions for court or field
Keeping time for relays
Number ball toss (Two teams whose members are numbered; when number is called, child must catch tossed ball before it touches ground to score a point for team.)

Music

Arranging varied lengths of pipe so tune can be played
Filling glasses with water enough so scale can be played
Raising or lowering of pitch by tightening or loosening a stretched wire
Locating pitch of sound made by tapping glass, block of wood, piece of metal, etc.

Art

Enlarging a small picture by means of squares (or vice versa)
Design a movable color wheel so that, as you spin parts, each section on top is smaller than the one below; label fractional parts.

Arithmetic

Design a multiplication (and division) wheel for the lower grades
Make flash cards; put answers on back. Work with a pal to flash the cards
Cut the numbers one to nine from a large calendar. Arrange them in three even columns

so that they equal 15 horizontally, vertically, and diagonally. Use colored spools, beads, and string to make an abacus.

Social Studies

Record daily weather data. Keep records of food fed hamsters during nutrition experiment; chart weight changes.

Clocks and maps. If every 15 degrees equal 1 hour of time, set the clock to the time it is now in London, San Francisco, and Chicago.

CURRICULUM CONSIDERATIONS FOR LOW ACHIEVERS

Both individual school systems and larger curriculum organizations are now turning their attention to the production of curriculum guidelines and materials for low achievers.

In a review of attempts of major U.S. cities to deal with "needs problems, and capabilities of big-city children," **Today's Education** reported on the work of five school systems. In this article, Rickey (126), a mathematics consultant to the Dade County Public Schools, Florida, reported effective results from the placement of junior high school students in four levels of mathematics instruction. Writing teams of teachers have produced a curriculum bulletin, **Guide of Objectives and Activities for Learning,** and **Success in Mathematics (Goals),** to help individual students progress. Many of their teachers of low achievers have developed self-instructional materials and techniques.

Longshore (126), secondary school mathematics consultant in the Kansas Public Schools, reported use of individualized instruction and flexible scheduling to attempt to meet the needs of low-achieving students. To accommodate the varied backgrounds of their students three Kansas City secondary schools have adopted flexible modular scheduling. Under this plan students meet in large groups for presentation of new materials by the teacher, for summaries of student reports, for speakers from industry, for instruction through audiovisual materials, or for testing. Laboratory periods provide for study with teachers or other students and for learning by discovery through manipulative devices or through games or puzzles. Open laboratories and library resource centers allow for independent study or for review. The formulation of performance objectives enables the student to know exactly what he is expected to learn.

The National Council of Teachers of Mathematics has produced materials for low achievers in grades five through eight. This project, **Experiences in Mathematical Ideas,** emphasizes the importance of "experiences (which) grow out of the physical environment of the students and are developed through an assortment of teaching strategies which frequently culminate in laboratory-oriented activities" (159,160).

An earlier N.C.T.M. Project, **Experiences in Mathematical Discovery,** was directed to the task of producing appropriate materials for ninth grade students in the 25th to 50th achievement percentile range. The writing committee agreed that such materials "should provide opportunities for genuine mathematical discovery at the maturity level of the students and should stimulate them to continue their study of mathematics in the regular sequence" (158). Units have been published by N.C.T.M. on the following topics: Formulas, Graphs, and Patterns; Properties of Operations with Numbers; Mathematical Sentences; Geometry; and Arrangements and Selections.

Another group, the Central Iowa Low Achievers Mathematics Project, has developed materials for students in grades seven through nine. Sixteen booklets of field-tested materials are available from the project at a cost of approximately $1.25 per booklet. A typical publication is **Gimmicks** (156) which is divided into three major areas—drill exercises, games and puzzles, and motivation of new concepts. The booklet is essentially a collection of ideas found to be successful by the teachers in the project. Other references for further resource material are provided. Other booklets available include **Measurements, ESP (Enrichment Student Projects), Probability,** and **Mathematics in Sports.**

Stenzel (42) attempts to capture the interest of her students by introducing topics in modular arithmetic, geometry, algebra, coordinate geometry, graphing and topology. She also encourages the use of games, projects, and gadgets as well as having the students move around the room, to work together and to use the chalk rather than scratchpaper.

Hoffman (19), in a description of the Jefferson County Public School's Program for low achievers, outlines some procedures designed to meet the need of the low achiever:

- Multiple Activities—All work is completed and evaluated during the class period. The student is provided with at least two changes of activity during a period.
- Electric Printout Calculators—One for every two students is provided.
- Local Business Problems—Contributed by local businessmen and presented to the student on letterhead paper.
- Flow-Charting—To aid students in analyzing an algorithm.
- Mathematics Laboratory Experiments—Performed with inexpensive and easily assembled physical apparatus.
- Laboratory Multisensory Aids—e.g., Standard audiovisual equipment, calculators, scales, slide rules, mathematical equipment and models, and much equipment for individual student use.
- Laboratory Involvement Projects—e.g., puzzles and kits for individual use.

In working with 66 high school dropouts in the Houston area, Kneitz and Creswell (65) were able to produce an average gain of 7 months' achievement in 2 months of instruction as well as noticeable changes in attitude, poise, and dress. Techniques and materials used included beginning instruction with materials which would allow students to succeed, use of individualized instruction, programed instruction booklets, SRA Computational Skills Kits, cross numbers puzzle kits, the Work Book Cyclo-Teacher, frequent changes of pace, filmstrips, math games, and real life problems.

The U.S. Office of Education Conference on Low Achievers in Mathematics (132) considered the need for the development of curricular materials appropriate for low achievers. Their recommendations provide the framework for the following guidelines to curriculum development.

1. Identify appropriate aims for instruction The USOE report emphasized the importance of developing mathematical understandings essential for vocational competence. Particular attention was paid to the potential of cooperative work-study programs which focus upon the achievement of skills demanded by families of jobs.

Mallory (70) outlined the following criteria for selecting course material:

- Every topic selected must definitely and positively contribute to the social, home, or community needs of the pupil.
- Every topic must be of such a nature and presented in such a way as to interest the pupil and convince him of its value to him; that is, it must be intrinsically interesting and of evident worth.
- The material must be concrete to him and within his ability to succeed with it.
- The material must be of such a type and presented in such a way as to develop desirable habits, attitudes, and appreciations.

In 1970, the National Council of Teachers of Mathematics recognized the need for identifying minimal competencies in mathematics for all students. The Council charged a group of educators concerned with teaching low achievers to draw up a list of basic mathematical competencies, skills, and attitudes essential for enlightened citizenship in contemporary society.

2. Make multidisciplinary input

Whitehead, in his book **Aims of Education,** has observed that "You may not divide the seamless coat of learning . . . there is one subject matter for education, and that is life, in all of its manifestations." He protests that "scraps of gibberish" are being taught "in the name of algebra." The point is clearly made, not only for low achievers, but perhaps more importantly for them, that what is taught in mathematics classes must bear firm relationships to what is taught in other classes and what goes on "out there" in the world. Such an approach to curriculum building, which appears to be revitalized today, suggests that persons representing a wide variety of background must participate in the decisionmaking concerning curriculum building.

The USOE report urged that "national leadership should initiate action to recruit a team of mathematicians, teachers, psychologists, and other experts to develop outlines of the mathematical content of courses for low achievers" (132). Other curriculum groups have attempted to involve community personnel (parents, youth workers, etc.) in the development of materials.

3. Maximize success

The low achiever has, by definition, experienced difficulty in learning mathematics. Hence, one of his greatest needs is to experience success. The Experiences in Mathematical Ideas Project (EMI) has said this well:

> Success breeds success. Yet too many mathematics classes offer many students little else but experiences of failure. EMI materials are designed to provide activities that will give even the slower students successful encounters with mathematics. This is done by breaking down concept into very small components and devising simple tasks that will carefully lead the student to a grasp of the basic ideas. The teacher using EMI materials will have to resist the urge to move too quickly through the experiences because they look "too easy." (159)

4. Individualize instruction

The USOE report (132) recommended that a variety of learning materials should be provided to meet the differing rates of learning. Attention should be directed toward their flexibility and ease of use in achieving the objectives of the program for the low achievers.

5. Change pace

Again, essentially by definition, the low achiever is able to give his attention to mathematics for only a short period of time. Teachers are aware, however, that for activities which have in themselves rewards of success or accomplishment, such as tinkering with an old car, or playing a musical instrument, this same student's span of attention may show a remarkable growth!

Many curriculum projects, such as Experiences in Mathematical Ideas, have recognized the need for accounting for this characteristic of some students:

> A change is as good as a rest. Perhaps more than any other subject in the curruculum, mathematics poses the risk of becoming routine and dull in the classroom. The prospects are particularly bleak for the youngster who day after day faces the same kind of tasks that have repeatedly meant failure for him in the past. EMI materials attempt to approach familiar mathematics from a fresh point of view. At the same time, the ultimate objective is to develop a more complete understanding of the fundamental mathematical concepts included in the set of units. (159)

6. Use varied approaches to learning

People appear to differ in their learning aptitudes. Some persons are verbally oriented, some are helped more by pictures. By and large, a helpful approach to learning appears to be to provide a variety of stimuli to the student.

One extensive project, "Program for Learning in Accordance with Needs" (PLAN), is a system of individualized instruction which uses currently available instructional materials. Units are developed which prescribe various learning activities by which the student may accomplish assigned objectives. Of particular interest is the recognition by the system developers that individuals differ in respect to their learning styles. For example, some youngsters may prefer to learn, and can learn better, through auditory means while others are more disposed to a ready approach (114).

7. Provide for evaluation

In order to maximize the usefulness of a set

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of curricular materials, developers should obtain feedback concerning the effectiveness of those materials. The USOE report stressed that such evaluation should be conducted in a variety of schools with different types of low achievers. Furthermore, provision for evaluation should be built into the overall administrative scheme of operation at an early stage of the curriculum development process. In general, attempts to evaluate a project after completion of writing and field trials are destined to less than optimal usefulness.

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ADMINISTRATIVE GUIDELINES FOR IMPLEMENTING NEW CURRICULUMS

The USOE report (132) emphasizes the importance of sound administrative policies and procedures for implementing new curriculum patterns for low achievers.

1. Recognize the key role of the school administrator
   - To initiate and provide staff and community orientation programs
   - To see that teachers of low achievers obtain due recognition for their important work
   - To involve nonschool persons and agencies in the development of programs to prepare low achievers for useful roles in society
   - To interpret the program to the community
2. Provide adequate course offerings at every grade level

Needs for appropriate curricular materials for low achievers exist at the elementary, junior, and senior high school as well as at the junior college levels. Furthermore, Brain (95) has the encouraging observation that:

> Research into the psychology of learning indicates that students of low ability can learn much more than they are learning at present. Interestingly, there is evidence that students of low ability can learn mathematical concepts and process at age 16 to 18 that they were not able to learn when they were younger.

3. Use appropriate grouping procedures

The USOE report encourages the use of grouping for purposes of instruction by gearing instruction toward the characteristics of each group (132).

4. Provide assistance to teachers

Teaching low achievers is time-consuming and demanding. Extra effort must be spent in such activities as developing and using varieties of special audiovisual and manipulative learning materials. The USOE report recommends that the administration provide for teachers of low achievers paraprofessional support such as teacher-aides. As such facilities as instructional materials centers and mathematics resource laboratories become more common, administrators may well contemplate the prospect of providing teachers of special groups with released time for consultative, tutorial, and conference sessions with selected groups of students. As Stenguist (129) has noted:

> One of the important functions of a board of superintendents is to so arrange working conditions that the rest of the personnel are provided with time and opportunity to carry on diagnostic education for all types of pupils.

5. Involve parents and community in school programs

Without the support of parents and community, the work of the school is virtually doomed to failure. Many projects for low achievers have attempted to bridge the gap by providing for parent involvement in the programs. This involvement ranges from "mathematics for parents" courses, literacy programs, to block meetings in the community at which problems and issues relating to the school are considered.

6. Carefully select teachers for low-achieving classes

A high level of competencies—warmth, insight, imagination, resourcefulness, knowledge of psychology, firm grasp of mathematics is required by the teacher of low achievers. The administrator has a great responsibility to assign suitable personnel to these critical staff positions.

As the USOE report points out, administrators could do a greater service to the cause of the low achiever by assuming responsibility for and taking initiative in informing teacher education colleges of the needs and requirements of teachers for such positions (132).

7. Provide inservice education programs

Shulte and Wells (127) have outlined activities for school principals to facilitate the professional growth of their mathematics staff.

- Establish a library, accessible to all teachers
- Arrange for attendance of teachers at appropriate inservice activities
- Help to ensure that the staff is effective in implementing new programs
- Ensure that teachers who are effectively using innovative practices receive recognition.

They also offer specific suggestions for the conduct of inservice programs:

- Use videotape for recording demonstration lessons. "All teachers benefit from seeing master teachers in action."
- Keep in mind that the demonstration class is primarily a showcase of excellent technique, not a sample of what takes place in a classroom day after day.
- The inservice presentations should not be too theoretical; they must present ideas that teachers can apply in the classroom in the immediate future.
- If manipulative aids are discussed, teachers should have these aids in their hands and the session may be conducted much the same as if the teachers were the students. It may be wise to facilitate this practice in using aids by grouping teachers in two's and three's.
- At the secondary school level, mathematics teachers are more often specialists in the subject, so the emphasis need not be so much on content of new materials as on techniques of teaching and on relating techniques to content. Without such emphasis, secondary teachers often use modern materials in a traditional fashion.
- Some schools have adopted a "buddy" system for new teachers where an experienced teacher is assigned to a new teacher, to help him or her adjust to the school and to the teaching situation.

8. Provide adequate guidance facilities

Many students who are having difficulty with mathematics also experience difficulties with other subjects, and may encounter social and emotional problems as well. The proper identification, analysis, and treatment of this host of difficulties requires the team efforts of teachers, counselors, psychologists, social worker, and other concerned specialists.

9. Promote an investigative point of view

Despite all that is being done to attack the problem of low achievement, little is known concerning basic issues. As programs and practices are implemented, school personnel are encouraged to seek answers to such questions as:

- What are the causes of learning difficulties in mathematics?
- How are particular learning difficulties dealt with?
- At what developmental stage of the child are particular difficulties more successfully dealt with?
- Which methods or modes of instruction are best for students with particular difficulties?
- What factors in the home, community, and school appear to foster or hinder mathematics achievement?
- What teacher characteristics relate best to high pupil achievement?

Summary

The identification of teaching resources for low achievers depends upon an analysis of student characteristics, teaching strategies, and mathematics curriculum content.

Social and emotional problems of low achievers are often reflected in low motivation, short interest spans, antisocial behavior, high absence rates, and goals based only on the immediate future. These characteristics suggest that appropriate resource materials should be constructed in small units with provisions for individualized work, and should emphasize success and self-worth—possibly by including an incentive or reward system.

A wide variety of learning aids is useful with low achievers. Computational aids such as tables and desk calculators help to avoid an area which has usually caused previous failure on the part of the low achiever. Other laboratory devices provide activity learning which is useful for improving and maintaining motivation levels of low achievers. An activity-learning approach provides an excellent vehicle for promoting the following teaching procedures that are particularly effective with low achievers:

- Employ repetition through spiraling.
- Introduce small segments of material at a time.
- Use language which the class is likely to understand.
- Dramatize the material.
- Individualize the problem assignments.
- Accurately "read" the problems.
- Provide variety within the class period.
- Use concrete approaches to the subject matter.
- Provide activities.
- Hold frequent reviews.
- Use praise freely.
- Display good student work.
- Build assignments which lead to success.
- Use diagnostic testing before and after teaching.
- Measure the student against himself.
- Correlate mathematics with other subjects whenever possible.
- Establish consistent classroom management policies.
- Use audiovisual techniques when possible.
- Try supervised study rather than homework.
- Grade work the day it is turned in.
- Do not insist on verbalization.
- Encourage the use of calculators and tables.

Resource materials should have clear objectives which have some relationship to the immediate needs of the learner. (Although one usually thinks of needs in practical terms, recreational needs where mathematics puzzles and games may be worthwhile should not be overlooked.) Content material should develop desirable habits, attitudes, and appreciations as well. To do this, materials must combine low difficulty levels with high mathematical standards.

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Annotated Bibliography of Mathematics Topics and Content Approaches Suitable for Low Achievers in Articles Appearing in **The Arithmetic Teacher** and **The Mathematics Teacher**

Primary Grades

Andrews, E. E., and Doyal L. Nelson, "Beginning Number Experiences and Structured Materials," **The Arithmetic Teacher,** 10:330-33, 1963.

The authors make a plea to present computational procedures through meaningful problem situations so as to avoid pupils merely learning computation per se. A question is raised concerning the use of Cuisenaire rods.

Bachrach, Beatrice, "Do Your First Graders Measure Up? (A Report of a Unit with Disadvantaged Learners)," **The Arithmetic Teacher,** 16:537-38, 1969.

A brief description of a unit on measurement which was used with disadvantaged first graders in Chicago.

Bridges, Raymond B., Jr., "Easily Made Arithmetic Aids," **The Arithmetic Teacher,** 10:507-8, 1963.

Details for constructing two simple devices for illustrating elementary number concepts. The first is a "block box" of wood, and the second consists of stringing 1-inch dowels on varying lengths of string.

Fisher, Alan A., "The Peg Board—A Useful Aid in Teaching Mathematics," **The Arithmetic Teacher,** 8:186-88, 1961.

The author presents ideas on how to use the pegboard as an aid in teaching perimeter and area, place value, commutative law for multiplication, squares of numbers, equal ratios, coordinate axes, and degrees in a polygon.

Harshman, Hardwick W., David W. Wells, and Joseph N. Payne, "Manipulative Materials and Arithmetic Achievement in Grade 1," **The Arithmetic Teacher,** 9:188-92, 1962.

The conclusions of this study, along with the authors' recommendations, should be of particular interest to teacher educators and all those interested in research.

Liedtke, W., and T. E. Kieren, "Geoboard Geometry for Preschool Children," **The Arithmetic Teacher,** 17:123-26, 1970.

This article has some specific suggestions and questions which could be of value to teachers working with very young children on a geoboard. These suggestions and questions are in the areas of familiar shapes, plane figures and segments.

Intermediate Grades

Probability and Statistics

Burt, Bruce, "Drawing Conclusions from Samples (An Activity for the Low Achiever), **The Arithmetic Teacher,** 16:539-41, 1969.

The author presents appropriate sampling activities which can be used in the intermediate grades. He also gives a short list of questions which teachers of probability at this level might find useful.

Grass, Benjamin A., "Statistics Made Simple," **The Arithmetic Teacher,** 12:196-8, 1965.

The author tells how his students made use of some simple statistics in his fourth-grade class to determine whether some students had extra sensory perception.

Wilkinson, Jack D., and Owen Nelson, "Probability and Statistics—Trial Teaching in Sixth Grade," **The Arithmetic Teacher,** 13:100–6, 1966.

The authors describe an experiment and make a number of recommendations concerning their experiment, which some teachers will find applicable to teaching low achievers.

Measurement

Smart, James R., and John L. Marks, "Mathematics of Measurement," **The Arithmetic Teacher,** 13:283-87, 1966.

The article discusses some of the fundamental mathematics of measurement. In addition, there are some examples which illustrate methods for teaching these topics.

Calculation

Mayer, Louise A., "The Scarbacus of Scarsdale Abacus," **The Arithmetic Teacher,** 2:159, 1955.

This one-page article describes a student-made counting device.

Travers, Kenneth J., "Computation: Low Achiever's Stumbling Block or Stepping Stone," **The Arithmetic Teacher,** 16:523-8, 1969.

The author presents a plea to deemphasize computation as well as a brief history of the mathematics laboratory movement in England. The article includes a discussion of the use of calculators and computers for low achievers.

Maloney, John P., "Arithmetic at the Primary Level," **The Arithmetic Teacher,** 4:112-18 1957.

The author discusses some of the philosophy and techniques used at the Boston Curriculum Center for Arithmetic in teaching primary students.

McKillip, William D., "Patterns—A Mathematics Unit for Three- and Four-Year Olds," **The Arithmetic Teacher,** 17:15-18, 1970.

The work described in this article could be used for slow learners at the primary level. The article describes how number concepts may be introduced by making use of blocks or tiles or two colors which are arranged to form various patterns.

Stern, Catherine, "The Concrete Devices of Structural Arithmetic, **The Arithmetic Teacher,** 5:119-30, 1958.

The teaching of structural arithmetic is discussed with explanations of how various models and devices can be used to allow students to discover the structure of our number system and its interrelationships.

University of Illinois Committee on School Mathematics, "Arithmetic with Frames," **The Arithmetic Teacher,** 4:119-24, 1957.

This article gives some ideas of the U.I.C.S.M. project staff concerning the informal teaching of some fundamental ideas in mathematics at the elementary level.

Winthrop, Henry, "Arithmetical Brain-Teasers for the Young," **The Arithmetic Teacher,** 14:42-3, 1967.

A number of brain teasers appropriate for elementary pupils (as well as others) are enumerated.

Yates, William E., "The Trachtenberg System as a Motivational Device," **The Arithmetic Teacher,** 13:677-8, 1966.

The author justifies the use of the Trachtenberg speed system of mathematics as a motivational device for slow learners as well as for all other ability levels.

Arithmetic

Green, Roberta, "A Color-Coded Method of Teaching Basic Arithmetic Concepts and Procedures," **The Arithmetic Teacher,** 17:231-3, 1970.

The author describes a method which she found helpful in teaching a fifth-grade boy severely retarded in arithmetic achievement. The procedure described helped the boy to understand and master basic arithmetic concepts and processes.

Haines, Margaret, "Modular Arithmetic," **The Arithmetic Teacher,** 9:122-6 1962.

A fairly simple introduction to modular arithmetic is given. Some useful arithmetic checks on addition, subtraction, multiplication, and division are explained using modulo 9 arithmetic.

Le Ganke, Lucille, "Let's Use our Checkers and Checkerboards to Teach Number Bases," **The Arithmetic Teacher,** 14:573-5, 1967.

Various aspects of teaching number bases by using a checkerboard and checkers are presented. The material might be used to motivate students.

Livingstone, Isobel L., "Live Models in Arithmetic?" **The Arithmetic Teacher,** 17:18-2 1970.

The author describes a method which she has found useful in the intermediate grades for putting some "life" in her arithmetic classes by getting the students involved.

Lyda, W. J., and Margaret D. Taylor, "Facilitating an Understanding of the Decimal Numeration System through Modular Arithmetic," **The Arithmetic Teacher,** 11:101-3, 1964.

The article describes lessons taught by the authors to determine whether changes occur in understanding the decimal numeration system when pupils receive instruction in modular arithmetic.

Prielipp, Robert W., "Finger Reckoning," **The Mathematics Teacher,** 61:42-3, 1968.

The reader is introduced to how to multiply by using his fingers.

Geometry

Fennell, Francis M., "Multiplication Football," **The Arithmetic Teacher,** 17:236-7, 1970.

A very simple motivational device is described which the author found useful in reviewing basic multiplication facts to intermediate grade children. The game could be modified for use at other levels.

Froelich, Effie, "An Investigation Leading to the Pythagorean Property," **The Arithmetic Teacher,** 14:5004, 1967.

The author describes some intuitive approaches for obtaining background notions related to the Pythagorean theorem. No formal proofs are required.

Smith, Lewis B., "Pegboard Geometry," **The Arithmetic Teacher,** 12:271-4, 1965.

Smith describes some activities which can be performed by making use of a geoboard. The activities are designed in part to help develop the student's intuition concerning geometric shapes.

Teegarden, Donald O., "Geometry via T-Board," **The Arithmetic Teacher,** 16:485-7, 1969.

The author describes the use of a so-called "T-Board" which he successfully used to teach perimeter and area concepts to fifth- and sixth-grade classes.

Wells, Peter, "Creating Mathematics with a Geoboard," **The Arithmetic Teacher,** 17:347-9, 1970.

Suggestions for use of the geoboard with lower-grade children are given. The author suggests some possible discussion questions for use with the geoboard.

Percentage

Hauck, Eldon, "Concrete Materials for Teaching Percentage," **The Arithmetic Teacher,** 1:9-12, 1954.

The author discusses two difficulties in teaching percentage. He explains how he makes use of a "percentage box" to help in overcoming these difficulties.

Recreation

Cook, Nancy, "Fraction Bingo," **The Arithmetic Teacher,** 17:237-9, 1970.

Described is a game used to help fourth, fifth, and sixth graders to better understand fractions and discover how to find other names for a fractional number.

Dohler, Dora, "The Role of Games, Puzzles, and Riddles in Elementary Math," **The Arithmetic Teacher,** 10:450, 1963.

The article is a description of games, puzzles, and riddles which are appropriate for elementary pupils.

Hunt, Martin H., "Arithmetic Card Games," **The Arithmetic Teacher,** 15:736-8, 1968.

Directions are presented on how to play five card games whose purpose is to enhance the pupil's facility with number facts. The games are war, fish, number trick, number scrabble, and number rummy. Some of these games might also be appropriate for primary children.

Jennings, Thomas J., "The Magic Box," **The Arithmetic Teacher,** 12:377-8, 1965.

The author suggests a way to teach elementary pupils about number patterns. The magic box referred to is what some textbook writers call a function machine.

Parker, Helen, "Seesaw Game," **The Arithmetic Teacher,** 10:449, 1963.

The "Seesaw Game" is used to help students develop facility in the four basic operations. Careful selection of content could make the game appropriate for almost any grade level.

Ruderman, Harry D., "The Greatest–A Game," **The Arithmetic Teacher,** 17:80-81, 1970.

The author describes a game which involves the random selection of numbers and is appropriate for children in the elementary grades. Four variations of the game are also included.

Willerding, Margaret F., "Codes for Boys and Girls," **The Arithmetic Teacher,** 2:23-4, 1955.

This article describes how ciphers can be used in the elementary classroom as a worthwhile activity. Two examples of ciphers are given although their "answers" are not included.

Winick, David F., "Arithmecode' Puzzle," **The Arithmetic Teacher,** 15:178-9, 1968.

The article is a description of a decoding exercise which involves the correct solution to a number of mathematics problems. The example given could be modified for use at almost any difficulty level.

Problem Solving

Scaeffer, Anne W., and Albert H. Mauthe, "Problem Solving with Enthusiasm–The Mathematics Laboratory," **The Arithmetic Teacher,** 17:7-14, 1970.

The authors describe some of their experiences with a mathematics laboratory in the fifth grade. They include 10 problems (well documented) which they use with children in the intermediate grades. They also include some children's reactions to the laboratory.

Junior High School

Fractions

Archbold, John C., "Measuring with Maps," **The Arithmetic Teacher,** 14:393-5, 1967.

The article contains ideas for utilizing maps and bulletin boards to help in teaching certain mathematics concepts.

Braunfeld, Peter, and Martin Wolfe, "Fractions for Low Achievers," **The Arithmetic Teacher,** 13:647-55, 1966.

A brief overview of U.I.C.S.M.'s "stretcher-shrinker" ideas for teaching fractions to culturally disadvantaged seventh graders.

Junge, Charlotte W., "A Game of Fractions," **The Arithmetic Teacher,** 13:494, 1966.

Describes a fraction game which can be played by two, three, or four children.

Zytkowski, Richard Thomas, "A Game with Fraction Numbers," **The Arithmetic Teacher,** 17:82-3, 1970.

The article describes an unusual way in which students can acquire practice in the basic operations with fractions. The author says the approach can also be used to teach ratio and sequence of numbers.

Integers

Fremont, Herbert, "Pipe Cleaners and Loops –Discovering How to Add and Subtract Directed Numbers," **The Arithmetic Teacher,** 13:568-72, 1966.

A novel way of helping students acquire facility in adding and subtracting signed numbers is explained using pipe cleaners.

Geometry

Hart, Alice G., "The Angle Mirror Indoors," **The Arithmetic Teacher,** 17:419-23, 1970.

The author describes a device for use in laboratory activities involving measurements.

Liedtke, Werner, "What Can You Do with a Geoboard," **The Arithmetic Teacher,** 16:491-3, 1969.

Liedtke suggests a number of pegboard activities in the areas of graphs, distance, perimeter, area, multiplication, division, fractions, and networks.

Woodby, Lauren G., "The Angle Mirror Outdoors," **The Arithmetic Teacher,** 17:298-300, 1970.

The article describes how to construct and use an angle mirror.

Patterns

Edmonds, George F., "Discovering Patterns in Addition," **The Arithmetic Teacher,** 16:245-8 1969.

Edmonds explains how he used a guided discovery approach to help primary teachers in Nairobi to find certain number patterns in addition. The patterns developed could be of interest to teachers of grades 3-8 and perhaps even beyond.

Peterson, John A., and Joseph Hashiasaki, "Patterns in Arithmetic," **The Arithmetic Teacher,** 13:209-12, 1968.

The authors present quite a few number and word patterns which almost any junior or senior high teacher could find useful as supplementary or interest material.

Tinti, Robert, "Mathematics Through Cardboard Carpentry (A unit for Low Achievers)," **The Arithmetic Teacher,** 17:209-10, 1970.

The article cites some of the mathematical values of such a unit for low achievers. A summary of six advantages resulting from the unit of study are listed. For further information on materials and tools used in cardboard carpentry, write to:

Cardboard Carpentry
Educational Development Center, Inc.
55 Chapel Street
Newton, Massachusetts 02160

Wahl, M. Stoessel, "We Made It and It Works! the Classroom Construction of Sundials," **The Arithmetic Teacher,** 17:301-4, 1970.

Detailed instructions are given for constructing a sundial. This activity provides a good opportunity for teaching some elementary geometry of the circle.

White, Donald E., "An Approach to Modern Mathematics Through Pascal's Triangle," **The Arithmetic Teacher,** 10:441-45, 1963.

The article tells how eighth-grade students were introduced to Pascal's Triangle via coin-flipping exercises.

Algebra

Biggs, Edith, "Trial and Experiment," **The Arithmetic Teacher,** 17:26-32, 1970.

Miss Biggs gives suggestions on how teachers might initiate activity learning in the areas of time, area, number patterns, along with some others, in their classroom.

Getty, G. A., "Uncle Ed Juggles with Figures," **The Mathematics Teacher,** 48:143-46, 1965.

The article describes a mathematical "stunt" performed with two arrays of digits. This activity would probably be of most interest to junior and senior high students.

Travers, Kenneth J., "Through Clouds of Failure into Orbit," **The Arithmetic Teacher,** 15:591-98, 1968.

The article discusses the need for teaching mathematics to all students, and appeals for a change in approach. There is also a section on teaching the concepts of variable.

Number Theory

Davies, Robert A., "Low Achiever Lesson in Primes," **The Arithmetic Teacher,** 16:529-32, 1969.

The author presents an unusual way, based upon the U.I.C.S.M. stretcher and shrinker concepts, of having low achievers "discover" the primes between 1 and 100.

Loftus, Sonja, "Fibonacci Numbers: Fun and Fundamentals for the Slow Learner," **The Arithmetic Teacher,** 17:204-8, 1970.

Morton, R. L., "Divisibility by 7, 11, 13, and Greater Primes," **The Mathematics Teacher,** 61:370-4, 1968.

This article presents the mechanics of and the rationale for the divisibility tests described in the title of the article.

Application

"A Job Survey as Class Motivation in General Math," **The Mathematics Teacher,** 43-318-20, 1950.

The article describes how students were led to conduct a job survey, and how it affected their work during the year.

Bragdon, Wallace M., "Rapid Mental Calculations," **The Arithmetic Teacher,** 12:369-70, 1965.

The author presents methods for multiplying whole numbers between 1 and 100 mentally.

Gibb, E. Glenadine, "Mathematics in Baseball," **The Mathematics Teacher**, 45:35-6, 1952.

The author provides suggestions as to how mathematics related to baseball can be used to stimulate the interest of intermediate grade students.

Holtan, Boyd, "Motivation and General Math Students," **The Mathematics Teacher**, 57:20-5, 1964.

This is a description of a project to improve mathematics achievement by presenting material in contexts which were interesting to general mathematics students.

Pyatt, Gladys, "Field Project in Junior High Math," **The Mathematics Teacher**, 38:327, 1945.

Teachers interested in doing field work with their classes will find this article valuable.

Recreation

Bradfield, Donald L., "Sparking Interest in the Mathematics Classroom," **The Arithmetic Teacher**, 17:239-42, 1970.

The author makes a point for introducing comedy, mystery, and drama into the mathematics classroom via certain types of problems. Several examples of problems are given to accomplish the above. Answers are given in some cases.

Cappon, John, Sr., "Easy Construction of Magic Squares for Classroom Use," **The Arithmetic Teacher**, 12:100-5, 1965.

Teachers will find this helpful in preparing magic squares. See other articles in this section for use of these squares.

Condron, Bernadine F., "Game To Review Basic Properties and Vocabulary," **The Arithmetic Teacher**, 12:227-8, 1965.

Some of the vocabulary or properties reviewed include the following: addend, factor, reciprocal, identity element and distributive and commutative properties.

Hewitt, Frances, "4 x 4 magic squares," **The Arithmetic Teacher**, 9:392-95, 1962.

The title describes the article. A brief bibliography is given as well as a magic square whose numbers are congruent, modulo 8.

Janicki, George, "Bizz-Buzz Game in Arithmetic," **The Arithmetic Teacher**, 3:28, 1956.

The author describes a very simple game which he used with fifth-, sixth-, and seventh-grade students to test their knowledge of the multiplication tables.

———, "Cross Figure Puzzle," **The Arithmetic Teacher**, 3:16, 1956.

This is a crossword puzzle involving numerals instead of words.

McCombs, Wayne E., "Four-by-Four Magic Square for the New Year," **The Arithmetic Teacher**, 17:79-80, 1970.

The author describes a way to construct four-by-four magic squares that will total any desired number.

Nies, Ruth H., "Classroom Experiences with Recreational Arithmetic," **The Arithmetic Teacher**, 3:90-3, 1956.

The article describes a number of games and puzzles which the author found useful with her sixth-grade class.

Ruderman, Harry D., "Nu-Tic-Tac-Toe," **The Arithmetic Teacher**, 12:571-2, 1965.

The article gives directions for playing a modified version of the Tic Tac Toe game. Rules can be learned, according to the author-inventor, in less than 1 minute.

Timmons, R. A., "Tic Tac Toe—A Mathematical Game for Grades 4-9," **The Arithmetic Teacher**, 14:506-8, 1967.

A modification of a familiar game is explained. The modified tic tac toe can be played by a whole class and involves some knowledge of how points are plotted on a rectangular coordinate system.

Willerding, Margaret F., "Cross-Number Puzzles," **The Arithmetic Teacher**, 4:221-223, 226, 1957.

These are crossword puzzles which involve numerals instead of words. The first puzzle is appropriate for primary grades, the second one for intermediate, and the third for junior high school.

Williams, Russell, "Bingtac," **The Arithmetic Teacher**, 16:310-11, 1969.

A description of the equipment and rules needed to play this game is given. The author used the game at the junior high level.

High School

Algebra

Greenholz, Sarah, "Successful Practices in Teaching Mathematics to Low Achievers in Senior High School," **The Mathematics Teacher,** 60:329-35, 1967.

A number of appropriate problems for low achievers in high school as well as some specific teaching techniques are enumerated.

Niechin, Arlene, and Robert Brower, "The Abacus—A New Use for an Old Tool," **The Arithmetic Teacher,** 6:314-16, 1959.

The authors describe how they made use of the abacus in their sixth grade class. Several activities with the abacus are explained as well as some benefits reaped by the students.

Geometry

Bauer, Marie L., "Projects for Plane Geometry," **The Mathematics Teacher,** 44:235-9, 1951.

The article contains a large number of topics suitable for projects or written or oral reports. Many of the topics are appropriate for low achievers and could help to stimulate their interest in math. A bibliography is also included.

Applications

Bell, Clifford, "What Every Teacher Should Know about the Uses of Mathematics," **The Mathematics Teacher,** 56:302-6, 1963.

The article presents a summary of what mathematics courses will be necessary or helpful in the field of higher learning which students may choose after their high school graduation. The article should be of particular interest to teachers and counselors.

Lankford, Francis G., Jr., "Practical Math Is Challenging to Students," **The Mathematics Teacher,** 48:401-5, 1955.

The author presents a case for the use of practical mathematics to stimulate students.

Morse, Jarvis M., "Math Teachers Train Thrifty Citizens," **The Mathematics Teacher,** 43:328-31, 1950.

This article explores many possible areas for the use of mathematics in finance and buying.

Myers, Sheldon S., ed., "Drawing a Circle with a Carpenter's Square," **The Mathematics Teacher,** 45:367, 1952.

The problem presented in this very short article would be an appropriate laboratory exercise.

Parker, Jean, "The Use of Puzzles in Teaching Mathematics," **The Mathematics Teacher,** 48:218-27, 1955.

The author presents a list of several factors justifying the use of puzzles in the classroom, and a number of puzzles and games for use primarily at the high school level. An extensive bibliography is included.

Peck, Lyman C., ed., "Pharmaceutical Arithmetic in the Home," **The Mathematics Teacher,** 46:438-39 plus, 1953.

The enterprising teacher would find this material very useful.

Weidemann, Charles, "Speed of Travel in Covered Wagon Days," **The Mathematics Teacher,** 46:101-2, 1953.

This article uses the "historical approach" in presenting a topic in an interest-context.

Recreation

Adkins, Bryce E., "Adapting Magic Squares to Classroom Use," **The Arithmetic Teacher,** 10:498-502, 1963.

The author presents two ways of adapting magic squares for classroom use. The article is for the reader who may not have mastered these patterns.

Andree, Richard V., "Cryptography as a Branch of Mathematics," **The Mathematics Teacher,** 45:503-9, 1952.

This article describes how ciphers could be used advantageously in the mathematics classroom, especially as a motivating device in general mathematics classes. Examples of ciphers are given as well as a rather extensive bibliography.

Carnahan, Walter, "If a Cricket Chirps One Hundred Times a Minute, How Fast Does an Ant Run?" **The Mathematics Teacher,** 45:58, 1952.

A simple little problem which could be used as an attention getter at almost any level from junior high up.

Gorts, Jeannie, "Magic Square Patterns," **The Arithmetic Teacher,** 16:314-16, 1969.

The author describes some magic square patterns which are useful in construction of magic squares.

Keepes, Bruce D., "Logic in the Construction of Magic Squares," **The Arithmetic Teacher,** 12:560-2, 1965.

Keepes makes use of some applications of logic and of our numeration system in constructing magic squares.

Schiele, Gerald D., "A Three-by-Five Card Plus an Opaque Projection Plus an Ice Pick," **The Arithmetic Teacher,** 16:533-35, 1969.

The author describes a moral teaching technique which he found useful in teaching remedial mathematics.

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